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STATUS REPORT

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National Aeronautics and Space Administration Research Grant No. NsG-412 to New York University

Ref.: SC NsG-412 Project Director: J. J. Stoker

Title: Non-Linear Elasticity

Research in the field of this grant has been continued along very similar lines as in the previous report. The principal workers were F. John, C. Sensenig, and some students. J. J. Stoker was on leave of absence.

F. John continued his investigations in the non-linear theory of perfectly elastic solids. In preparation are a number of papers dealing with:

1) A proof that "harmonic materials" are exactly those for which "irrotational" initial states always lead to irrotational motions. (This result was announced at the IUTAM meeting in Tiflis, 1963.) Here the term "harmonic materials" refers to those with a strain energy function of the form

$$W = f(s_1) + as_2 + bs_3$$

where s_1, s_2, s_3 are the elementary symmetric functions of the principal strains, $f(s_1)$ is an arbitrary function, and a, b arbitrary constants. The motion is irrotational if given by $x_i = \partial F(X_1, X_2, X_3, t)/\partial X_i$, where x_i and X_k are coordinates in the strained and unstrained states respectively.

2) A uniqueness proof for the first boundary value problem under the sole assumption that the maximum strains are sufficiently small. (The proof relies heavily on the John-Nirenberg lemma.)

3) A revision of the work on quasi-isometric mappings (contained in the IMM-NYU report 336, January, 1965) for publication in the Communications for Pure and Applied Mathematics.

Further research plans include a deeper penetration of shell theory, and of deformations with bounded strain energy. The Ph.D. thesis of J. Roseman on St. Venant's principle has been completed, and its author has accepted an Assistant Professorship at the University of Wisconsin. The thesis, inter alias, rounds off the results of Toupin in a significant way.

F. John gave an invited talk on the role of inequalities in the theory of elasticity at the SIAM meeting held in May at the Courant Institute.

C. Sensenig continued his work on buckling and on estimating elastic stresses and displacements.

The non-linear buckling of a circular plate has been studied for displacements which are polynomials of degree n in the undeformed distance to the middle surface. Numerical calculations have been made for $n=1,2,3$ for a wide range of plate thicknesses and vertical deflections. The results for $n=2,3$ agree to about three significant digits over the entire range of parameters used thus far. For $n=1$, the results differ from those for $n=2,3$ when the vertical deflection is small as was predicted analytically, but the results for $n=1$ appear to approach asymptotically those for $n=2,3$ as the vertical deflection increases. He expects to continue the above calculations covering a wider range of parameters and boundary conditions and also to make comparisons with the results of the von-Karman-Föppl theory.

Sensenig has derived estimates for derivatives of displacements and for stresses and their derivatives for a non-linear shell theory with the same assumption on the displacements as above. Estimates have been obtained for plates using the three dimensional non-linear theory. This work differs from that of F. John in that the estimates are given in terms of displacements rather than stresses. In many respects, however, this work closely follows that of F. John.

An attempt is being made to obtain similar estimates in the shell theory referred to above. This has been done for $n = 1$, but for $n \geq 2$ difficulties have been encountered. It is expected that the case $n \geq 2$ can be worked out in time.

BIBLIOGRAPHY

F. John, "Estimates for the Derivatives of the Stresses in a Thin Shell and Interior Shell Equations", Comm. Pure Appl. Math., 18(1965), pp. 235-267.

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J. Roseman, "On the Principle of St. Venant", Thesis. Parts of this paper are to be published in the Archive for Rational Mechanics and Analysis.

C. Sensenig, "Non-Linear Theory for the Deformation of Pre-Stressed Circular Plates and Rings", Comm. Pure Appl. Math., 18(1965), pp. 147-161.